08-02-2007

Applications of Quantum Physics

Mobile phones should be switched off during the exam

1. Binding Energies

Consider a silicon 2+ ion: $Si^{2+}(1s^2 2s^2 2p^6 3s^2)$.

For reference, the binding energy of H(*1s*) is 13.6 eV.

a) Calculate the ionization potential (in eV) of this ion. The quantum defect is δ=1.91.

b) Calculate the effective charge experienced by the least-bound electron.

c) The total binding energy of the two 3s electrons together is 78.6 eV. Calculate the quantum defect describing the binding energy of the 3s electron in $Si^{3+}(1s^2 2s^2 2p^6 3s)$.

d) Why is the quantum defect calculated at b) larger, equal or smaller than the one given at a).

2. Configurations, Terms, States, and Hund's rules

Zirconium has the following ground electronic configuration:
$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^2 5s^2$.

a) Use LS coupling to determine all possible terms and states associated with this electronic configuration

b) Indicate which one is the ground state and sketch the binding energy sequence of the terms and states assuming that the Hund's rules apply to all terms.

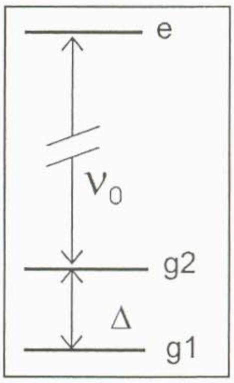
3. Doppler-free saturation spectroscopy

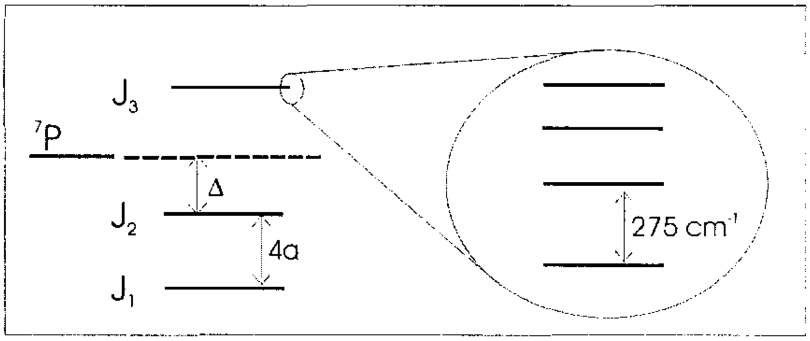
a) Briefly describe and/or sketch the method of Doppler-free saturation spectroscopy.

The figure depicts the electronic structure of the atoms in the gas cell.

b) Sketch the Doppler-free saturation spectrum.

c) If Δ = 400 MHz and $\nu_0 = 8\text{x}10^{14}$ Hz, calculate for the peaks and/or dips in the spectrum the velocities of the atoms that give rise to these peaks and/or dips.





4. Fine and Hyperfine structure

The figure shows the fine structure of a septet P term and the hyperfine structure of the upper fine state (J_3).

a) Determine the fine structure quantum numbers J_1,J_2 and J_3.

b) The fine splitting between J_1 and J_2 is 4a. Calculate the shift Δ (in units of a) of J_2 with respect to the "unsplit" ^7P term.

c) The zoom in shows the hyperfine structure of J_3. Deduce the nuclear spin of the atom.

d) Determine the F quantum numbers of the hyperfine states and calculate the hyperfine constant (in units of cm^{-1}).

5. Quantum interrogation

On the night before the examination you become curious about the questions (and answers) of the examination which are kept in a large box in the cellars of Nijenborgh. This box is also one of the possible sleeping places of a dog. Secretly you enter Nijenborgh. The problem is that you don't know whether the dog is sleeping in the box and you can't take the risk of waking the dog if it is indeed sleeping in the box, because then it will start barking and alarm security.
One way or the other all you can do is to pass an item of food into a small flap in the box. If the food is something uninteresting to dogs, like a salad, there will be no reaction — the dog will just keep slumbering peacefully, oblivious to the food. But if the food smells delicious like meat, the aromas will awaken the dog, which will begin barking.
It seems you are stuck. If you stick a salad into the box, you don't learn anything, because you can't tell the difference between a sleeping dog and no dog in the box. If you stick meat into the box, you surely learn whether the dog is in there, but only because it will wake up and alarm security. Luckily, the physics department is the proud owner of a quantum food processor (QFP) that can turn food into a superposition of salad and meat, which allows you to tackle the problem through quantum physics.

Food = $a|\text{salad}\rangle + b|\text{meat}\rangle$ and $|a|^2 + |b|^2 = 1$

You start out with a salad. Using the QFP you apply a $\Pi/2$ pulse to the wavefunction.
Pass the *Food* (wavefunction, don't look at it, let the dog look) into the box. The "box" remains quiet. Retract the *Food* from the box and put it into the QFP and apply once again a $\Pi/2$ pulse.

a) After the second $\Pi/2$ pulse give the wavefunction *Food* if there is no dog
b) After the second $\Pi/2$ pulse give the wavefunction *Food* if the dog is in the box
c) If the dog is in the box what is the probability for you to know
d) Do your changes of finding out (in secret) whether the dog is in the box improve if you use 3 pulses of length $\Pi/3$ ($\Pi/3$ – *Food* into box, *Food* out of box – $\Pi/3$ – *Food* into box, *Food* out of box – $\Pi/3$ – detection)?
e) Determine a pulse sequence to find out with a probability of more than 99% whether the dog is inside the box or not.